A Theory of Independent Fuzzy Probability for System Reliability

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***Abstract*—Fuzzy fault trees provide a powerful and computationally efficient technique for developing fuzzy probabilities based on independent inputs. The probability of any event that can be described in terms of a sequence of independent unions, intersections, and complements may be calculated by a fuzzy fault tree. Unfortunately, fuzzy fault trees do not provide a complete theory: many events of substantial practical interest cannot be described only by independent operations. Thus, the standard fuzzy extension (based on fuzzy fault trees) is not complete since not all events are assigned a fuzzy probability. Other complete extensions have been proposed, but these extensions are not consistent with the calculations from fuzzy fault trees. In this paper, we propose a new extension of crisp probability theory. Our model is based on n independent inputs, each with a fuzzy probability. The elements of our sample space describe exactly which of the n input events did and did not occur. Our extension is complete since a fuzzy probability is assigned to every subset of the sample space. Our extension is also consistent with all calculations that can be arranged as a fault tree. Our approach allows the reliability analyst to develop complete and consistent fuzzy reliability models from existing crisp reliability models. This allows a comprehensive analysis of the system. Computational algorithms are provided both to extend existing models and develop new models. The technique is demonstrated on a reliability model of a three-stage industrial process.**

***Index Terms*—Fuzzy fault trees, fuzzy probability, fuzzy sets, independence.**

I. Introduction

Many system reliability models require (as input) the probabilities of a number of independent events. Often these probabilities can be estimated from data or theory, but sometimes choosing probabilities for input is difficult. This work is part of an ongoing study in high-consequence surety analysis. Many of the factors of interest come from traditionally nonmathematical areas of research such as estimating the probability of a terrorist attack, compliance with safety practices, or a flawed design of a safety system. Other factors are too expensive or dangerous to measure experimentally. Instead, expert opinion is used to provide these probabilities, but these estimates are rarely precise. Fuzzy sets and possibility theory provide a tool for describing and analyzing these uncertain quantities.

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An uncertain parameter $F \in \Re$ may be assigned a fuzzy membership function $\overline{F}(y)$: $\Re \rightarrow [0, 1]$, which is the membership function of a fuzzy set $\overline{F}$. Then the possibility that F is in a set S is designated by $\prod_F(S)$ and

$$\prod_F (S) = \sup_{y\in S} \overline{F}(y).$$

This is the sense in which we describe uncertainty in the probability of an event A.

In this paper, $\overline{P}_A$ is a fuzzy set describing uncertainty in the crisp probability $P(A)$ and $\overline{P}_A(y)$: $[0, 1] \rightarrow [0, 1]$. Fuzzy fault trees provide a method for developing fuzzy probabilities based on independent fuzzy inputs $\overline{P}_A$ [1]. The probability of any event that can be described in terms of a sequence of independent unions, intersections, and complements may be calculated by a fuzzy fault tree. Unfortunately, we show below that some events of substantial practical interest cannot be described only by independent operations: Standard fuzzy fault trees do not provide a complete theory. Thus, the standard fuzzy extension (based on fuzzy fault trees) is not complete since not all events are assigned a fuzzy probability. Zadeh proposed another extension which is complete [11], but his extension is shown (in our context) to be inconsistent with the calculations from fuzzy fault trees. For recent developments of fuzzy probability, see [2]–[5]. Walley and de Cooman, in particular, discuss completeness and consistency in a more general setting. See also Cai's work on system failure engineering [6] and other discussions of fuzzy fault trees [7]–[10].

Here we develop a new extension of crisp probability theory based on n independent inputs, each with a fuzzy probability. The elements of our sample space describe exactly which of the n input events did and did not occur. This extension will be shown to be both complete and consistent.

II. Independent Calculations and Fuzzy Fault Trees

Throughout this paper, we use the bar notation $\overline{P}_A$ to indicate a fuzzy set representing probability of A, the notation $\overline{P}_A(y)$ to indicate the corresponding upper semicontinuous membership function and $\overline{P}_A^\alpha = \{y\colon \overline{P}_A(y) \geq \alpha\}$ to indicate the corresponding α cuts. A convex fuzzy set $\overline{P}_A$ has special structure: each α cut is a closed and convex subset of $\Re$. We see for a convex fuzzy probability that each α cut can be written as a closed interval with $\overline{P}_A^\alpha = [P_{A1}^\alpha, P_{A2}^\alpha]$. This assumption of convexity is equivalent to assuming that the membership function has a single mode. Earlier work with independent fuzzy probabilities relied on this (often quite reasonable) assumption of convexity, but our work will be more general. Following the lead of most fuzzy models, all fuzzy sets here are required to have nonempty $\alpha = 1$ cut. This property follows here from normality.

Consider independent events $A_1, A_2, A_3, \cdots, A_n$ with estimated fuzzy probabilities $\overline{P}_{A_1}, \overline{P}_{A_2}, \cdots, \overline{P}_{A_n}$, which will be used in a reliability model. Following the lead of Tanaka *et al.* [1] and many others, our concept of independence is crisp. We assume the underlying probabilities for $A_1, A_2, A_3, \cdots, A_n$ are independent in the conventional (crisp) sense. Our goal is to build a fuzzy probability theory to describe the probabilities of various unions, intersections, and complements of these sets. To this end, we follow the standard approach of Tanaka *et al.* [1] and first build fuzzy intersections of independent events.

If events A_i are independent, then for crisp probabilities we have

$$P(A_i \cup A_j) = P(A_i) + P(A_j) - P(A_i)P(A_j)$$

and

$$P(A_i \cap A_j) = P(A_i)P(A_j).$$

Using the usual extension principle, we define the fuzzy independent union and intersection as

$$\overline{P}_{A_i \cup A_j}(y) = \sup_{y = p_i + p_j - p_i p_j} \min[\overline{P}_{A_i}(p_i), \overline{P}_{A_j}(p_j)] \quad (1)$$

and

$$\overline{P}_{A_i \cap A_j}(y) = \sup_{y = p_i p_j} \min[\overline{P}_{A_i}(p_i), \overline{P}_{A_j}(p_j)]. \quad (2)$$

Complements of fuzzy probabilities are similarly defined by

$$\overline{P}_{A_i^c}(y) = \sup_{y = 1 - p_i} \overline{P}_{A_i}(p_i) = \overline{P}_{A_i}(1 - y). \quad (3)$$

We then have the following familiar properties:

$$\begin{aligned}
\overline{P}_{A_i \cup A_j} &= \overline{P}_{A_j \cup A_i} \\
\overline{P}_{A_i \cap A_j} &= \overline{P}_{A_j \cap A_i} \\
\overline{P}_{(A_i \cup A_j) \cup A_k} &= \overline{P}_{A_i \cup (A_j \cup A_k)} \\
\overline{P}_{(A_i \cap A_j) \cap A_k} &= \overline{P}_{A_i \cap (A_j \cap A_k)} \\
\overline{P}_{(A_i \cup A_j)^c} &= \overline{P}_{A_i^c \cap A_j^c} \\
\overline{P}_{(A_i \cap A_j)^c} &= \overline{P}_{A_i^c \cup A_j^c}.
\end{aligned} \quad (4)$$

This third formula is DeMorgan's law and extends in the obvious way to

$$\begin{aligned}
\overline{P}_{(A_1 \cup A_2 \cup \cdots \cup A_k)^c} &= \overline{P}_{A_1^c \cap A_2^c \cap \cdots \cap A_k^c} \\
\overline{P}_{(A_1 \cap A_2 \cap \cdots \cap A_k)^c} &= \overline{P}_{A_1^c \cup A_2^c \cup \cdots \cup A_k^c}.
\end{aligned} \quad (5)$$

If the fuzzy probabilities are convex, we have the relationships between endpoints of the α-cut intervals

$$\begin{aligned}
&[P^\alpha_{A_i \cup A_j 1}, P^\alpha_{A_i \cup A_j 2}] \\
&\quad = [P^\alpha_{A_i 1} + P^\alpha_{A_j 1} - P^\alpha_{A_i 1} P^\alpha_{A_j 1}, P^\alpha_{A_i 2} + P^\alpha_{A_j 2} - P^\alpha_{A_i 2} P^\alpha_{A_j 2}]
\end{aligned} \quad (6)$$

and

$$[P^\alpha_{A_i \cap A_j 1}, P^\alpha_{A_i \cap A_j 2}] = [P^\alpha_{A_i 1} P^\alpha_{A_j 1}, P^\alpha_{A_i 2} P^\alpha_{A_j 2}]. \quad (7)$$

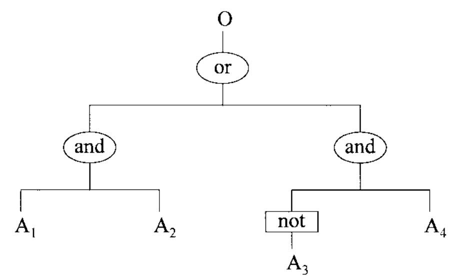


Fig. 1. A fuzzy fault tree that maintains independence.

Unfortunately, the distributive laws fail. Straightforward application of the above formulas show

$$\begin{aligned}
\overline{P}_{A_i \cup (A_j \cap A_k)} &\neq \overline{P}_{(A_i \cup A_j) \cap (A_i \cup A_k)} \\
\overline{P}_{A_i \cap (A_j \cup A_k)} &\neq \overline{P}_{(A_i \cap A_j) \cup (A_i \cap A_k)}.
\end{aligned} \quad (8)$$

This formula fails because $(A_i \cup A_j)$ and $(A_i \cup A_k)$ are not independent so (4) cannot be applied.

As we see in (8), care must be used in organizing calculations to maintain independence. This is usually done by describing calculations as a tree structure. This viewpoint was naturally assumed in several papers on fuzzy fault trees [1], [12]–[16]. To illustrate this concept, consider the example tree diagram in Fig. 1. This diagram contains three varieties of nodes: unions, intersections, and complements. At the nodes, fuzzy input probabilities are combined according to the formulas in (1)–(3). As long as the tree only feeds upward and each node has only one output, independence is maintained. Because of DeMorgan's laws in (5), we can develop fault trees using only unions and intersections (but no complements) or only intersections and complements (but no unions). Thus, several somewhat different approaches to fault trees are in fact equivalent when the standard extensions in (1)–(3) are used.

Unfortunately, many problems do not easily fit into a straightforward tree structure, with each node having only one output. In our investigations, certain factors (such as terrorism risk) influence many different events so that construction of independent trees is problematic. As we will see in the next section, other problems also occur.

III. Completeness

The representation of some sets can be rearranged to allow use of (1)–(3). For example, in (8), since $A_i \cup A_j$ is not (necessarily) independent of $A_i \cup A_k$, we could simply define

$$\overline{P}_{(A_i \cup A_j) \cap (A_i \cup A_k)} = \overline{P}_{A_i \cup (A_j \cap A_k)}. \quad (9)$$

Now A_j and A_k are independent so we can correctly calculate $\overline{P}_{A_j \cap A_k}$ using (2). Since A_i is independent of $A_j \cap A_k$, we can apply (1) to calculate $\overline{P}_{A_i \cup (A_j \cap A_k)}$. Unfortunately, unraveling such relationships can be very difficult in complex models. Of greater concern is the fact that not all possible fuzzy probabilities can be calculated by rearranging into a calculation that maintains independence.

For example, a listing of all possible independent calculations easily shows that $\overline{P}_{(A_i^c \cap A_j) \cup (A_i \cap A_j^c)}$ may not be rearranged to allow calculation by independence formulas.

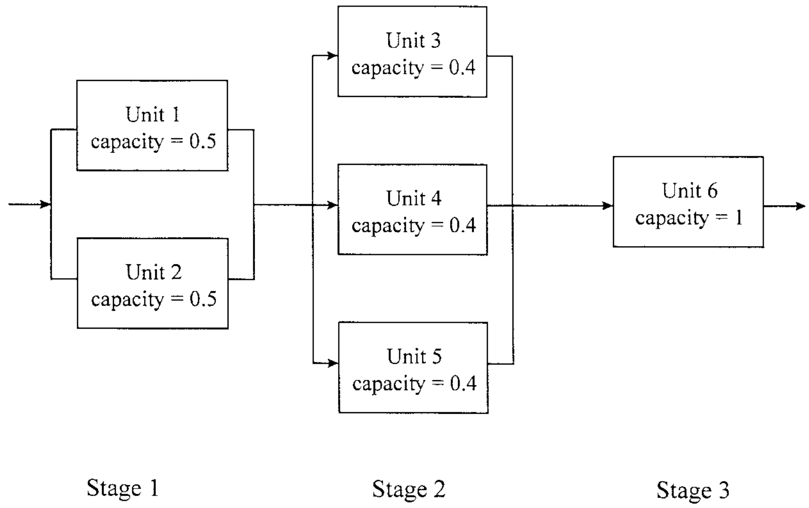


Fig. 2. A three-stage industrial process to be modeled with fuzzy probabilities.

Consider two independent system components numbered i and j. If event A_i indicates that i is operational and A_j indicates that j is operational, then $\overline{P}_{(A_i^c \cap A_j)\cup(A_i \cap A_j^c)}$ is the fuzzy probability that exactly one of the two components is operational. The inability of (1)–(3) to calculate such probabilities is a serious limitation in reliability applications. This problem with dependence was first recognized by Cooper [15]. It was also discussed briefly by the authors in [17].

This limitation is illustrated by the example we use in this paper. Consider the three-stage manufacturing process shown in Fig. 2. This diagram shows the flow of an industrial process through three stages. Stage one may be performed by two redundant units, each with a throughput capacity of 0.5 items per second. If both units 1 and 2 are operational, stage one has a throughput capacity of one item per second. If only one of the two units is operational, the stage one throughput is 0.5 items per second. If neither unit 1 nor unit 2 are operational, the throughput capacity of stage one is zero. This viewpoint may be used to build the throughput capacity of the entire process, with the capacity of stage one limiting the possible flow through stage two, and so on. Let A_i be the event that unit i is operational. Assume the process has repairable (or replaceable) independent units and that the process has been in operation long enough to approximately reach stationarity. Then $p_i = P(A_i)$ is the stationary readiness coefficient of unit i [18]. Letting T be the process throughput capacity, we can calculate the steady-state distribution of T as

$$\begin{aligned} &P(T=1)\\ &\quad = P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6)\\ &P(T=0.8)\\ &\quad = P(A_1 \cap A_2 \cap ((A_3 \cap A_4 \cap A_5^c) \cup (A_3 \cap A_4^c \cap A_5)\\ &\qquad \cup (A_3^c \cap A_4 \cap A_5)) \cap A_6) \end{aligned} \tag{10}$$

and so on. Possible values of T are $\{0,\ 0.4,\ 0.5,\ 0.8,\ 1.0\}$. Calculation of the distribution of T follows easily when the stationary readiness coefficients are crisp; our goal is to study this process with fuzzy readiness coefficients. To calculate the fuzzy probability $\overline{P}_{T=0.5}$, we must calculate the fuzzy probability that exactly one of units 1 and 2 is functional. Unfortunately, as discussed in the proceeding paragraph, this fuzzy probability cannot be modeled using (1)–(3). Several other "gaps" occur in the fuzzy reliability model of the system.

Clearly, many important fuzzy probabilities cannot be reached by the standard independence formulas in (1)–(3). To understand what sets are missing, we should more carefully specify the probability space of interest in our reliability problem.

Traditional probability theory is built with the concept of a probability triple: [S, $\mathbf{F}$, $P(\cdot)$] with sample space S describing all possible outcomes of the experiment, a collection $\mathbf{F}$ of subsets of S meeting the requirements of a σ field and a probability measure $P(\cdot)$ that assigns a number to each member of $\mathbf{F}$. We build our fuzzy triplet from independent sets $A_1,\ A_2,\ A_3,\ \cdots,\ A_n$.

Definition: The sample space S_n based on n independent events $\{A_1,\ A_2,\ \cdots,\ A_n\}$ is the set of 2^n distinct elements

$$S_n = \{s_1,\ s_2,\ \cdots,\ s_{2^n}\}$$

of the form

$$s = \bigcap_{i=1}^{n} B_i, \qquad \text{with } B_i = A_i \text{ or } B_i = A_i^c.$$

For the remainder of this paper, the notation A_i will be used to indicate the independent events from which S_n is defined.

For example, for $n = 3$

$$\begin{aligned} S_3 = \{&A_1 \cap A_2 \cap A_3,\ A_1^c \cap A_2 \cap A_3,\ A_1 \cap A_2^c \cap A_3\\ &A_1 \cap A_2 \cap A_3^c,\ A_1^c \cap A_2^c \cap A_3,\ A_1^c \cap A_2 \cap A_3^c\\ &A_1 \cap A_2^c \cap A_3^c,\ A_1^c \cap A_2^c \cap A_3^c\}. \end{aligned}$$

These are the basic elements in our reliability problem. Each describes exactly which of the events A_i did and did not occur.

Note that S_n has a finite number of elements, so our sample space is discrete. A fuzzy probability theory, in keeping with both our needs and the structure of crisp probability theory for discrete sample spaces, should assign a probability for every subset of S_n.

Definition: The σ field $\mathbf{F_n}$ over which fuzzy probabilities are defined is the collection of all subsets of S_n, including the empty set ϕ and S_n itself.

We then have a natural definition for completeness and the following result follows from the discussion above.

Definition: A fuzzy probability theory is called complete if it assigns a fuzzy probability to every set in the collection $\mathbf{F_n}$.

Proposition 1: The independent fuzzy probability theory $\overline{P}_B$ is incomplete.

To develop a complete theory, we need to characterize those sets that can be reached through (1)–(3).

Proposition 2: Suppose A and B are subsets of S_n. Using the notation $P(A_i) = p_i$, A and B are independent if and only if we can write $P(A) = f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})$ and $P(B) = f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}})$ for disjoint subsequences $\{r_i\}$ and $\{s_i\}$ of $\{1, 2, \cdots, n\}$.

Proof: In crisp probability theory, completeness is achieved for discrete probability spaces by use of additivity: if sets C and D are disjoint, $P(C \cup D) = P(C) + P(D)$. When this is applied to the discrete sample space $S_n = \{s_1, s_2, \cdots, s_{2^n}\}$ we calculate the crisp probability of a subset A of S by

$$P(A) = P\left(\bigcup_{s_i \in A} \{s_i\}\right) = \sum_{s_i \in A} P(\{s_i\}). \tag{11}$$

Thus, every subset of the discrete S_n is broken into a finite union of disjoint singleton sets $\{s_i\}$ and probabilities for all subsets of S_n are inherited from the list of probabilities for each element of S_n.

We will find it helpful to chose a specific ordering of the elements s_k in S_n. Consider, for k in $\{1, 2, 3, \cdots, 2^n\}$, the n-digit binary number $\beta_k = (\beta_{k1}, \beta_{k2}, \cdots, \beta_{kn})$ representing $k-1$

$$k - 1 = \sum_{i=1}^{n} \beta_{ki} 2^{i-1}. \tag{12}$$

With this binary number, we can pick an explicit ordering of the s_k

$$s_k = \bigcap_{i=1}^{n} B_{ki}$$

with

$$B_{ki} = \begin{cases} A_i^c, & \text{when } \beta_{ki} = 0 \\ A_i, & \text{when } \beta_{ki} = 1. \end{cases} \tag{13}$$

Now, letting $P(A_i) = p_i$, we have

$$P(\{s_k\}) = \prod_{i=1}^{n} p_i^{\beta_{ki}} (1 - p_i)^{(1-\beta_{ki})}$$

and for any set $A \subset S_n$

$$P(A) = P\left(\bigcup_{s_k \in A} \{s_k\}\right) = \sum_{s_k \in A} \prod_{i=1}^{n} p_i^{\beta_{ki}} (1 - p_i)^{(1-\beta_{ki})}. \tag{14}$$

Note that this is a multivariate polynomial in the p_i and especially note that the highest power of each of the p_i is one.

Now consider the statement of proposition 2. If $\{r_i\}$ and $\{s_i\}$ are disjoint, then independence is clear. If A and B are independent, then

$$\begin{aligned} P(A \cap B) &= P(A)P(B) \\ &= f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}}) \\ &\quad \times f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}}). \end{aligned}$$

If $\{r_i\}$ and $\{s_i\}$ are not disjoint, at least one of the p_i occurs linearly (when the p_j for $i \neq j$ are held fixed) in both $f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_n})$ and $f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_n})$. Thus, this p_i occurs quadratically in $P(A \cap B)$. This contradicts the result in (14) so, in fact, the $\{r_i\}$ and $\{s_i\}$ must be disjoint subsequences of $\{1, 2, \cdots, n\}$. □

With this characterization of independence we can describe those sets for which a fuzzy probability can be calculated using (1)–(3). Consider a set $B \subset S_n$ that can be constructed through independent operations. Note that the set B must be constructed through the binary operations of $\cup$ and $\cap$ as well as through use of complements. Because of the DeMorgan's relationships in (4), the expression for B can be written in terms of $\cup$, $\cap$, and sets A_1, A_1^c, A_2, A_2^c, $\cdots$, A_n, A_n^c without further need for complements. We can view such a B by working backward through a binary tree of union and intersection operations

$$B = \begin{cases} A_i, & \text{for some } i \text{ or} \\ A_i^c, & \text{for some } i \text{ or} \\ B_{11} \square_1 B_{12} \end{cases} \tag{15}$$

with B_{11} independent of B_{12} and

$$\square_1 = \cup \text{ or } \cap.$$

Similarly, for B_{11} and B_{12}

$$B_{11} = \begin{cases} A_i, & \text{for some } i \text{ or} \\ A_{i,}^c & \text{for some } i \text{ or} \\ B_{21} \square_{21} B_{22} \end{cases}$$

with B_{21} independent of B_{22}

$$\square_{21} = \cup \text{ or } \cap$$

and

$$B_{12} = \begin{cases} A_i, & \text{for some } i \text{ or} \\ A_i^c, & \text{for some } i \text{ or} \\ B_{23} \square_{22} B_{24} \end{cases}$$

with B_{23} independent of B_{24} and

$$\square_{22} = \cup \text{ or } \cap. \tag{16}$$

Assuming this decomposition has no trivial components (such as union with the empty set ϕ or intersection with S_n) from Proposition 2 we see that the fork in the binary tree splits a subsequence of $\{1, 2, \cdots, n\}$ into two pieces each with length of at least one. Thus, there are only a finite number of branches before the tree terminates with an A_i or A_i^c at each branch.

Definition: We say that event $B \subset S_n$ can be organized as an independent calculation if it can be written in the binary tree pattern of (15) and (16).

Definition: For an event $B \subset S_n$ which can be organized as an independent calculation, we define $\overline{P}_B$ as the fuzzy probability theory resulting from repeated application of (1)–(3).

IV. Zadeh's Linguistic Probabilities and Consistency

Now we must build the definition of fuzzy probability for subsets of S_n from the given fuzzy probabilities $\overline{P}_{A_1}, \overline{P}_{A_2}, \cdots, \overline{P}_{A_n}$. Following Zadeh [11], we can define an extension of (11). Consider a proper subset B of a sample space $S_n = \{s_1, s_2, \cdots, s_{2^n}\}$ with $B = \{\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_k\}$ where $\tilde{s}_i$ are the elements in S_n that are in B. Using a superscript Z to indicate Zadeh's extension we define

$$\overline{P}_B^Z(y) = \left(\sup_{\substack{y=x_1+x_2+\cdots+x_k\\ x_1+x_2+\cdots+x_k\le 1}}\right) \min[\overline{P}_{\{\tilde{s}_1\}}(x_1), \overline{P}_{\{\tilde{s}_2\}}(x_2), \cdots, \overline{P}_{\{\tilde{s}_k\}}(x_k)]. \tag{17}$$

The inequality in the sup is a result of the interactivity of crisp probabilities, since

$$\sum_{i=1}^{2^n} P(\{s_i\}) = 1.$$

Each $\overline{P}_{\{s_i\}}(\cdot)$ is calculated from $\overline{P}_{A_1}, \overline{P}_{A_2}, \cdots, \overline{P}_{A_n}$ using independence and (1)–(3). Similarly, for S we have

$$\overline{P}_S^Z(y) = \left(\sup_{\substack{y=x_1+x_2+\cdots+x_{2^n}\\ x_1+x_2+\cdots+x_{2^n}=1}}\right) \min[\overline{P}_{\{s_1\}}(x_1), \overline{P}_{\{s_2\}}(x_2), \cdots, \overline{P}_{\{s_{2^n}\}}(x_{2^n})]. \tag{18}$$

This formulation does provide a fuzzy probability for every subset of S_n. Unfortunately, (17) and (18) are not consistent with the calculations in (1)–(3). This is easily illustrated with a simple example using $n = 2$. Then $S_2 = \{A_1 \cap A_2, A_1^c \cap A_2, A_1 \cap A_2^c, A_1^c \cap A_2^c\}$. Note that $A_1 = \{A_1 \cap A_2\} \cup \{A_1 \cap A_2^c\}$. Consider convex fuzzy probabilities $\overline{P}_{A_1}$ and $\overline{P}_{A_2}$ represented by α cuts $[P_{A_1 1}^\alpha, P_{A_1 2}^\alpha]$, and $[P_{A_2 1}^\alpha, P_{A_2 2}^\alpha]$. Then, from (3) and (7)

$$\begin{aligned}&[P_{A_1\cap A_2 1}^\alpha, P_{A_1\cap A_2 2}^\alpha]\\ &\quad = [P_{A_1 1}^\alpha P_{A_2 1}^\alpha, P_{A_1 2}^\alpha P_{A_2 2}^\alpha]\end{aligned} \tag{19}$$

$$\begin{aligned}&[P_{A_1\cap A_2^c 1}^\alpha, P_{A_1\cap A_2^c 2}^\alpha]\\ &\quad = [P_{A_1 1}^\alpha(1-P_{A_2 2}^\alpha), P_{A_1 2}^\alpha(1-P_{A_2 1}^\alpha)].\end{aligned} \tag{20}$$

Using (19) and (20) and following (17)

$$\begin{aligned}\overline{P}_{A_1}^{Z\alpha} &= [P_{A_1 1}^\alpha P_{A_2 1}^\alpha + P_{A_1 1}^\alpha(1-P_{A_2 2}^\alpha)\\ &\qquad P_{A_1 2}^\alpha P_{A_2 2}^\alpha + P_{A_1 2}^\alpha(1-P_{A_2 1}^\alpha)]\\ &= [P_{A_1 1}^\alpha - P_{A_1 1}^\alpha(P_{A_2 2}^\alpha - P_{A_2 1}^\alpha)\\ &\qquad P_{A_1 2}^\alpha + P_{A_1 2}^\alpha(P_{A_2 2}^\alpha - P_{A_2 1}^\alpha)]\\ &\ne [P_{A_1 1}^\alpha, P_{A_1 2}^\alpha].\end{aligned} \tag{21}$$

Clearly, since $(P_{A_2 2}^\alpha - P_{A_2 1}^\alpha) \ge 0$, the uncertainty in $\overline{P}_{A_1}^Z$ somehow grows when another independent event A_2 is considered in the model. We have an inconsistency in the Zadeh model derived from independent probabilities.

We can now formally define a consistent model.

Definition: An independent fuzzy probability theory is called consistent if it is a direct extension of (1)–(3). By this, we mean that all calculations that may be organized as independent calculations may be calculated with (1)–(3).

From our example above, we immediately have the following.

Proposition 3: Zadeh's linguistic probabilities $\overline{P}_B^Z$, when derived from independent fuzzy probabilities $\overline{P}_{A_i}$, are inconsistent.

V. A Complete and Consistent Formulation of Independent Fuzzy Probabilities

As an alternative to Zadeh's approach, we consider a different extension of (11). Consider a reliability model built in terms of the independent fuzzy probabilities $\overline{P}_{A_i}$, $i = 1, 2, \cdots, n$, for sample space S_n. Using (for crisp probabilities) the definition $p_i = P(A_i)$, we see for $B \subset S_n$ that

$$\begin{aligned}P(B) &= P\left(\bigcup_{s_i\in B}\{s_i\}\right)\\ &= \sum_{s_i\in B} P(\{s_i\})\\ &= f_B(p_1, p_2, \cdots, p_n)\end{aligned} \tag{22}$$

for a function $f_B(\cdot)$. Thus, the crisp probability of every $B \subset S_n$ can be written uniquely as a function $f_B(\cdot)$ in terms of $p_1, p_2, \cdots, p_n$. For the empty set ϕ we have $f_\phi(p_1, p_2, \cdots, p_n) = 0$ and for the sample space we have $f_{S_n}(p_1, p_2, \cdots, p_n) = 1$. We use these functions to build our extension of (1)–(3). We can now define our extension for $B \subset S_n$.

Definition: For $B \subset S_n$, the extension of independent fuzzy probabilities is

$$\overline{P}_B^E(y) = \sup_{y=f_B(p_1, p_2, \cdots, p_n)} \min[\overline{P}_{A_1}(p_1), \overline{P}_{A_2}(p_2), \cdots, \overline{P}_{A_n}(p_n)] \quad (23)$$

with $P(B) = f_B(p_1, p_2, \cdots, p_n)$ when $P(A_i) = p_i$. If, for a fixed y, the set $\{(p_1, p_2, \cdots, p_n): y = f_B(p_1, p_2, \cdots, p_n)\}$ is empty, we take $\overline{P}_B(y) = 0$. The function $f_B(\cdot)$ is defined in (22).

Note that as long as the $\overline{P}_{A_i}$ are normal fuzzy sets, then the $\overline{P}_B$ is normal. We also see that

$$\overline{P}_{S_n}^E(y) = \begin{cases} 0 & y < 1 \\ 1 & y = 1 \end{cases}$$

and

$$\overline{P}_{\phi}^E(y) = \begin{cases} 1 & y = 0 \\ 0 & y > 0. \end{cases}$$

From this definition of fuzzy probability we get the following result.

Proposition 4: Suppose A and B are independent subsets of S_n. Then

a)
$$\overline{P}_{A\cap B}^E(y) = \sup_{y=pq} \min[\overline{P}_A^E(p), \overline{P}_B^E(q)] \quad (24)$$

b)
$$\overline{P}_{A\cup B}^E(y) = \sup_{y=p+q-pq} \min[\overline{P}_A^E(p), \overline{P}_B^E(q)] \quad (25)$$

c)
$$\overline{P}_{A^c}^E(y) = \sup_{y=1-p} \overline{P}_A^E(p) = \overline{P}_A^E(1-y). \quad (26)$$

Proof: We will demonstrate only part (a); the other parts follow in the same way. Consider

$$\overline{P}_{A\cap B}^E(y) = \sup_{y=f_{A\cap B}(p_1, p_2, \cdots, p_n)} \cdot \min[\overline{P}_{A_1}(p_1), \overline{P}_{A_2}(p_2), \cdots, \overline{P}_{A_n}(p_n)].$$

Noting Proposition 2 and the normality of the $\overline{P}_{A_i}$

$$\begin{aligned}\overline{P}_{A\cap B}^E(y) &= \left(\sup_{y=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}}) f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}})}\right) \min[\overline{P}_{A_1}(p_1), \overline{P}_{A_2}(p_2), \cdots, \overline{P}_{A_n}(p_n)] \\ &= \left(\sup_{y=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}}) f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}})}\right) \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \\ &\qquad \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}}), \overline{P}_{A_{s_1}}(p_{s_1}) \\ &\qquad \overline{P}_{A_{s_2}}(p_{s_2}), \cdots, \overline{P}_{A_{s_{n_B}}}(p_{s_{n_B}})\Big].\end{aligned}$$

Here, $\{s_i\}$ and $\{r_i\}$ are disjoint subsequences of $\{1, 2, \cdots, n\}$. Then

$$\begin{aligned}&\overline{P}_{A\cap B}^E(y) \\ &= \left(\sup_{y=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})q}\right)\left(\sup_{q=f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}})}\right) \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}}), \\ &\qquad \overline{P}_{A_{s_1}}(p_{s_1}), \overline{P}_{A_{s_2}}(p_{s_2}), \cdots, \overline{P}_{A_{s_{n_B}}}(p_{s_{n_B}})\Big] \\ &= \left(\sup_{y=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})q}\right) \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}}) \\ &\qquad \left(\sup_{q=f_B(p_{s_1}, p_{s_2}, \cdots, p_{s_{n_B}})}\right) \min\Big[\overline{P}_{A_{s_1}}(p_{s_1}), \overline{P}_{A_{s_2}}(p_{s_2}), \cdots, \overline{P}_{A_{s_{n_B}}}(p_{s_{n_B}})\Big]\Big] \\ &= \left(\sup_{y=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})q}\right) \cdot \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}}), \\ &\qquad \overline{P}_B^E(q)\Big].\end{aligned}$$

Continuing this pattern of calculations

$$\begin{aligned}&\overline{P}_{A\cap B}^E(y) \\ &= \left(\sup_{y=pq}\right)\left(\sup_{p=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})}\right) \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}}), \\ &\qquad \overline{P}_B^E(q)\Big] \\ &= \left(\sup_{y=pq}\right) \min\Big[\sup_{p=f_A(p_{r_1}, p_{r_2}, \cdots, p_{r_{n_A}})} \min\Big[\overline{P}_{A_{r_1}}(p_{r_1}), \overline{P}_{A_{r_2}}(p_{r_2}), \cdots, \overline{P}_{A_{r_{n_A}}}(p_{r_{n_A}})\Big] \\ &\qquad \overline{P}_B^E(q)\Big] \\ &= \left(\sup_{y=pq}\right) \min\big[\overline{P}_A^E(p), \overline{P}_B^E(q)\big].\end{aligned}$$

□

We now have our main result.

Proposition 5: The extension $\overline{P}_B^E$, when derived from independent fuzzy probabilities $\overline{P}_{A_i}$ is both consistent and complete.

Proof: Completeness is immediate since $\overline{P}_B^E$ is defined for every subset of S_n. To see consistency, we note that $\overline{P}_{A_i}^E = \overline{P}_{A_i}$. From the representation of B as a nested set of independent operations, we establish consistency by repeatedly applying Proposition 4. □

The reader should note that completeness and consistency in Proposition 5 hold for the model discussed in this paper; in [2] and [5] we see that consistency cannot be guaranteed in a more general setting.

VI. Implementing Convex Fuzzy Probabilities

For the special (but important) case of convex fuzzy probabilities, we can represent a membership function in terms of the left and right endpoints of its α cuts. The following theorem provides an important computational tool.

Proposition 6: Suppose for $A \subset S_n$ that the crisp probability relationship is $P(A) = f_A(p_1, p_2, \cdots, p_n)$ and $P(A_i) = p_i$, as in (22). Assume $\overline{P}_{A_i}$ are convex. Then $\overline{P}_A^E$ is also convex and $\overline{P}_A^{E\alpha} = [P_{A1}^{E\alpha}, P_{A2}^{E\alpha}]$ with

$$\begin{aligned} P_{A1}^{E\alpha} = \min_{1 \le k \le 2^n} f_A\Big(&(P_{A_1 1}^{\alpha})^{\beta_{k1}} (P_{A_1 2}^{\alpha})^{(1-\beta_{k1})}, \\ &(P_{A_2 1}^{\alpha})^{\beta_{k2}} (P_{A_2 2}^{\alpha})^{(1-\beta_{k2})}, \cdots \\ &(P_{A_n 1}^{\alpha})^{\beta_{kn}} (P_{A_n 2}^{\alpha})^{(1-\beta_{kn})} \Big) \end{aligned} \tag{27}$$

and

$$\begin{aligned} P_{A2}^{E\alpha} = \max_{1 \le k \le 2^n} f_A\Big(&(P_{A_1 1}^{\alpha})^{\beta_{k1}} (P_{A_1 2}^{\alpha})^{(1-\beta_{k1})}, \\ &(P_{A_2 1}^{\alpha})^{\beta_{k2}} (P_{A_2 2}^{\alpha})^{(1-\beta_{k2})}, \cdots \\ &(P_{A_n 1}^{\alpha})^{\beta_{kn}} (P_{A_n 2}^{\alpha})^{(1-\beta_{kn})} \Big). \end{aligned} \tag{28}$$

Proof: Consider (23). From the definition of $f_A(\cdot)$ in (22), we see that $y \in \overline{P}_A^{E\alpha}$ if and only if there is a point $(p_1, p_2, \cdots, p_n)$ in the hyper-rectangle $R = [P_{A_1 1}^{\alpha}, P_{A_1 2}^{\alpha}] \times [P_{A_2 1}^{\alpha}, P_{A_2 2}^{\alpha}] \times \cdots \times [P_{A_n 1}^{\alpha}, P_{A_n 2}^{\alpha}]$ with $y = f_A(p_1, p_2, \cdots, p_n)$. Now let

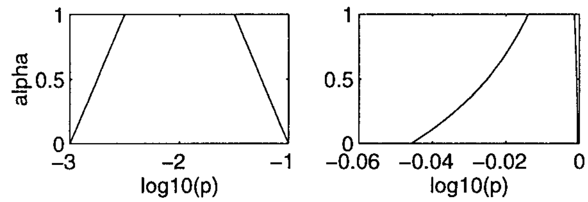
$$l = \min_R f_A(p_1, p_2, \cdots, p_n).$$

Note from (14) that $f_A(p_1, p_2, \cdots, p_n)$ is affine in p_i when the p_j are held fixed for $j \ne i$. Then the minimum must occur on a vertex of the hyper-rectangle R. The binary numbers β_{ki} make it convenient to represent the search over the vertices.

$$\begin{aligned} l = \min_{1 \le k \le 2^n} f_A\Big[&(P_{A_1 1}^{\alpha})^{\beta_{k1}} (P_{A_1 2}^{\alpha})^{(1-\beta_{k1})}, \\ &(P_{A_2 1}^{\alpha})^{\beta_{k2}} (P_{A_2 2}^{\alpha})^{(1-\beta_{k2})}, \cdots \\ &(P_{A_n 1}^{\alpha})^{\beta_{kn}} (P_{A_n 2}^{\alpha})^{(1-\beta_{kn})} \Big]. \end{aligned} \tag{29}$$

If we evaluate $f_A(p_1, p_2, \cdots, p_n)$ at the vertex of R which leads to the minimum l, we get from (22) the right-hand side of (27). Thus, the right-hand side of (27) is in $\overline{P}_A^{E\alpha}$. If y is less than the right-hand side of (27), then $y \notin \overline{P}_A^{E\alpha}$. Following the same argument for

$$r = \max_R f_A(p_1, p_2, \cdots, p_n)$$



(a) (b)

Fig. 3. (a) The fuzzy idleness coefficient and (b) the readiness coefficient for a single unit.

we see that the right-hand side of (28) is in $\overline{P}_A^{E\alpha}$ and $y \notin \overline{P}_A^{E\alpha}$ when y is greater than the right-hand side of (28). A simple continuity argument then shows that (27) and (28) hold. □

Note in (27) and (28) that any (computationally efficient) crisp probability model may be used to implement $f_A(\cdot)$. Thus, Proposition 6 provides a simple technique to generalize existing crisp reliability models.

We can implement the extended fuzzy probability formula in a fully general and straightforward way. Consider subsets A, B, and C of S_n. Associate with A the vector $(a_1, a_2, \cdots, a_{2^n})$ with

$$a_i = \begin{cases} 1, & s_i \in A \\ 0, & s_i \notin A. \end{cases} \tag{30}$$

For subsets B and C of S_n define similar vectors $(b_1, b_2, \cdots, b_{2^n})$ and $(c_1, c_2, \cdots, c_{2^n})$. Then, for $C = A \cup B$

$$c_i = \max(a_i, b_i). \tag{31}$$

For $C = A \cap B$

$$c_i = \min(a_i, b_i).$$

For $C = A^c$

$$c_i = 1 - a_i.$$

Finally, for each A_i, we can associate a vector $(a_{i1}, a_{i2}, \cdots, a_{i2^n})$, and

$$a_{ij} = \begin{cases} 1, & \beta_{ji} = 1 \\ 0, & \beta_{ji} = 0. \end{cases} \tag{32}$$

This formula makes use of the orderings of the s_k in (13). Note in these relationships that no assumptions of independence are required. Using (31) and (32), complicated set relationships represented as either formulas or diagrams can be easily implemented. We then have the following formula.

Proposition 7: Suppose $A \subset S_n$ is represented by the vector in (30) and the elements s_k of S_n are represented by the β_{ki} in (12) and (13). Assume $\overline{P}_{A_i}$ are convex. Then $\overline{P}_A^E$

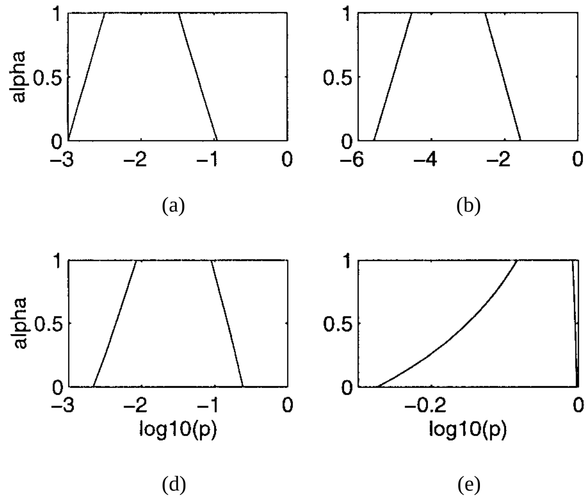


Fig. 4. The resulting fuzzy probabilities for the process throughput (a) $T = 0$, (b) $T = 0.4$, (c) $T = 0.5$, (d) $T = 0.8$, and (e) $T = 1$.

is also convex and $\overline{P}_A^{E\alpha} = [P_{A1}^{E\alpha}, P_{A2}^{E\alpha}]$ with

$$P_{A1}^{E\alpha} = \min_{1\le k\le 2^n} \sum_{j=1}^{2^n} a_j \prod_{i=1}^{n} \times \left[(P_{A_i1}^{\alpha})^{\beta_{ki}}(P_{A_i2}^{\alpha})^{(1-\beta_{ki})}\right]^{\beta_{ji}} \times \left[1-(P_{A_i1}^{\alpha})^{\beta_{ki}}(P_{A_i2}^{\alpha})^{(1-\beta_{ki})}\right]^{(1-\beta_{ji})} \qquad (33)$$

and

$$P_{A2}^{E\alpha} = \max_{1\le k\le 2^n} \sum_{j=1}^{2^n} a_j \prod_{i=1}^{n} \times \left[(P_{A_i1}^{\alpha})^{\beta_{ki}}(P_{A_i2}^{\alpha})^{(1-\beta_{ki})}\right]^{\beta_{ji}} \times \left[1-(P_{A_i1}^{\alpha})^{\beta_{ki}}(P_{A_i2}^{\alpha})^{(1-\beta_{ki})}\right]^{(1-\beta_{ji})}. \qquad (34)$$

Proof: This proposition follows immediately from Proposition 6 and from (22):

$$P_A = f_A(p_1, p_2, \cdots, p_n) = \sum_{j=1}^{2^n} a_j \prod_{i=1}^{n} [p_i]^{\beta_{ji}}[1-p_i]^{(1-\beta_{ji})}.$$

□

Although (31)–(34) are not particularly efficient in terms of required storage and computations, they are very easy to implement and are practical to use for n less than approximately ten. Note that only the fuzzy probability of the final answer needs be calculated. Intermediate fuzzy calculations are not required. Also, these equations allow general convex $\overline{P}_{A_i}$ instead of restricting inputs to triangular, trapezoidal, or some other form. For much larger problems, a more careful organization of calculations that takes advantage (when possible) of independence and Propositions 5 and 6 would be required.

To demonstrate the technique, we consider the three stage process discussed above and illustrated in Fig. 2. To demonstrate the calculations, the event $T = 0.8$ will be discussed. Note from (10), using independence and additivity, that

$$P(T=0.8) = f_{T=0.8}(p_1, \cdots, p_6) = p_1p_2(p_3p_4(1-p_5) + p_3(1-p_4)p_5 + (1-p_3)p_4p_5)p_6.$$

This is the expression used in Proposition 6. To simplify the illustration, all six independent units are assumed to have the fuzzy readiness coefficient shown in Fig. 3. Fig. 4 shows the resulting fuzzy probabilities for $T = 0$, $T = 0.4$, $T = 0.5$, $T = 0.8$, and $T = 1.0$. These fuzzy probabilities describe the long-term performance of the industrial process.

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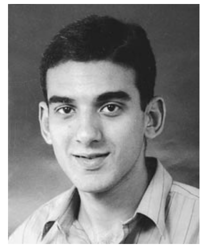

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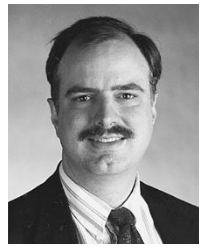

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